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A Two Non-Identical Unit Parallel System with Random Appearance and Disappearance of Repairman

Abstract

The paper deals with a two non- identical units (unti-1 and unit-2) parallel redundant system with waiting time for the appearance of repairman. It means that the repair facility is not available always with the system. A single repairman is to be available to repair a failed unit who appears and disappears from the system randomly only if there is no failed unit in the system. Once the repairman is engaged in the repair of a failed unit, he does not disappear from the system and completes the repair of all the units that fail during his stage in the system.

Keywords: Regenerative point, Reliability, Mean Time To System Failure, Busy period of repairman, Net Expected Profit.

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Introduction

In real life situations, the systems are becoming complex day by day due to automation and ever increasing demands of society. The improvement in effectiveness in respect of reliability, availability and net expected profit has therefore become important in recent years. To enhance the reliability of such types of systems introduction of redundancy is one of the method. The various authors Kishan, Jain and Sherbenyl[8,9], Goel and Murari [1], Gupta etal.[2,7,5] have analyzed the system models by taking standby redundancy. Sometimes when standby system takes some significant time to start the operation due to imperfect or slow switching device, it will be wisable to use redundant unit in parallel form with the main unit, so that the system does not fail if the main operative unit fails.

Keeping this fact in view some authors [3-4] have analyzed two unit parallel system models by taking different concepts. In this paper we have analyzed a two non-identical unit parallel system model assuming constant failure and general repair rates.

Using regenerative point technique, the following economic measures of the system effectiveness are obtained:

1. Reliability and Mean Time To System Failure (MTSF).
2. Point-wise and steady-state availabilities of the system and expected up time of the system during time (0, t).
3. Expected busy period of repairman during (0, t).
4. Net expected profit earned by the system during time (0, t) and in steady- state.

Aim of the Paper

The purpose of the present paper is to devoted the concept of random appearance and disappearance of repairman in two non-identical unit parallel system with failure and repair times of each unit as correlated random variables having their joint distribution as bivariate exponential.

Model Description and assumptions

1. The system comprises of two non-identical units (unit-1 and unit-2) in parallel configuration. Initially both units are operative.
2. Each unit of the system has two modes: normal (N) and total failure (F).
3. A single repairman is available to repair a failed unit who appears in and disappears from the system randomly only if there is no failed unit in the system.
4. Once the repairman is engaged in the repair of a failed unit, he does not disappear from the system and completes the repair of all the units that fail during his stage in the system.
5. The priority in repair is being given to unit-1 over the repair of unit-2. The restarted repair of unit-2(after interception) is of pre-

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emptive type i.e. time already spent in the repair of unit-2 goes to waste.

6. Each repaired unit works as good as new.
7. All the failure time distributions and disappearance time distribution of repairman are exponential with different parameters. The distributions of time to repair of unit-1, unit-2 and distribution of time to appearance of repairman are taken general with different cdfs.

Review of Literature.

This paper presents the analysis of a two-unit parallel system in which the server appears and disappears from the system at random. The failure and repair times of each unit are assumed to be correlated and their joint density is taken as a bivariate exponential.

Notations and states of the system

(a) Notations

α_1 : Constant failure rate of unit-1.

α_2 : Constant failure rate of unit-2.

β : Constant rate of disappearance of repairman.

$G_1(.)$: cdf of repair time of unit-1.

$G_2(.)$: cdf of repair time of unit-2.

H(.) : cdf of time to appearance of repairman.

(b) Symbols used for the states of the system

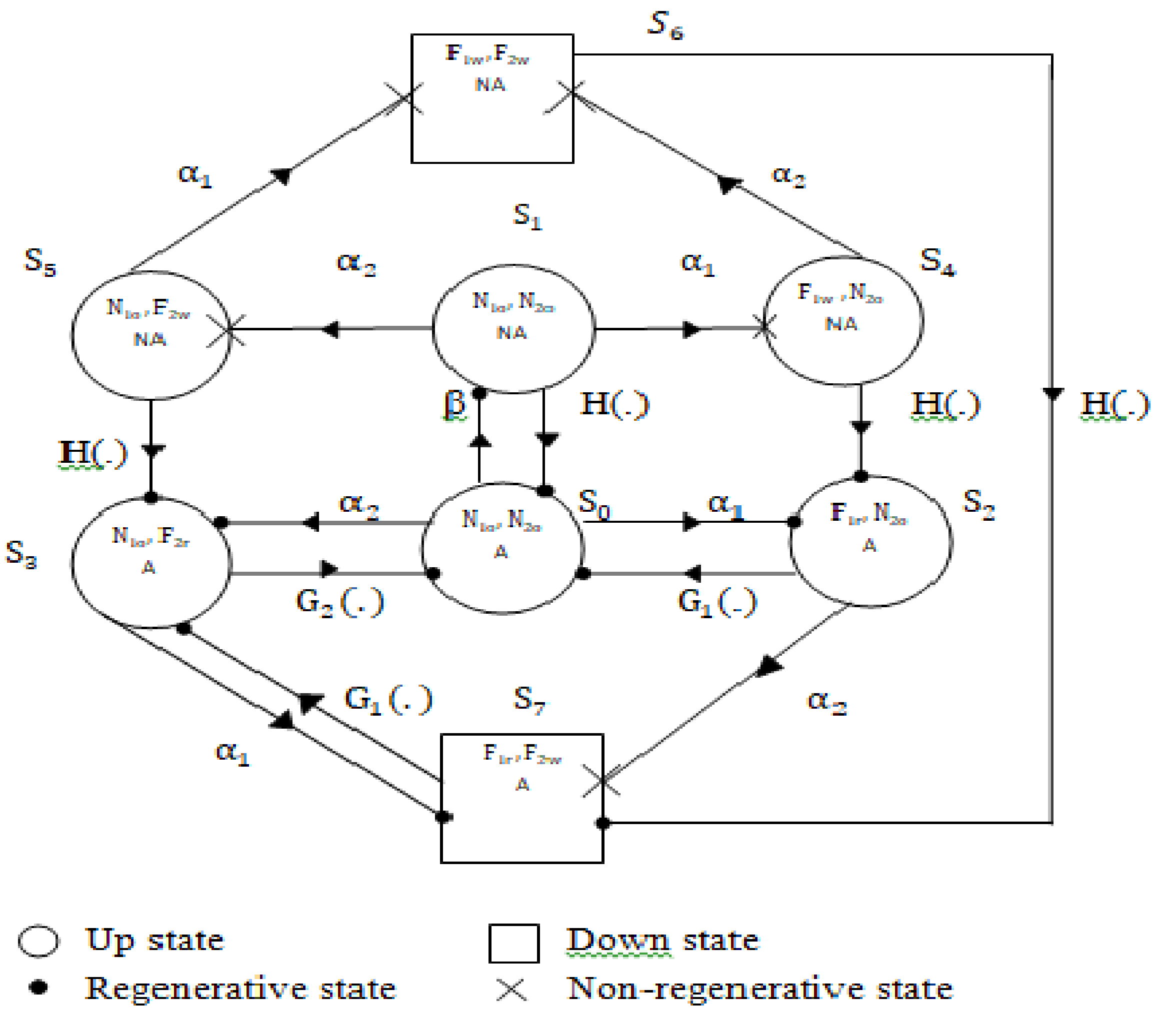
N_{io} : Unit-i is in N mode and operative.

F_{ir} : Unit-i is in F mode and under Repair.(i=1,2)

F_{iw} : Unit-i is in F mode and wait for Repair.(i=1,2)

A, NA : The repairman is available, not available with the system.

Transition Diagram



Here the epochs of entrance into the states S_4 from S_1, S_5 from S_1, S_7 from S_2 and S_6 from S_4 and S_5 are non-regenerative points whereas all other entrance points are regenerative.

Transition Probabilities and Sojourn Times

(a) The steady- state unconditional and conditional transition probabilities can be obtained as follows

$p_{01} = \frac{\beta}{\alpha_1 + \alpha_2 + \beta}$

$p_{02} = \frac{\alpha_1}{\alpha_1 + \alpha_2 + \beta}$

$p_{03} = \frac{\alpha_2}{\alpha_1 + \alpha_2 + \beta}$

$p_{10} = \tilde{H}(\alpha_1 + \alpha_2)$

$p_{12}^{(4)} = \tilde{H}(\alpha_2)$- $\tilde{H}(\alpha_1 + \alpha_2)$

$p_{13}^{(5)} = \tilde{H}(\alpha_1)$- $\tilde{H}(\alpha_1 + \alpha_2)$

$p_{17}^{(4,6)} = 1$ -$\tilde{H}(\alpha_2)$ - $\frac{\alpha_2}{\alpha_1+\alpha_2}$ + $\frac{\alpha_2}{\alpha_1+\alpha_2}\tilde{H}(\alpha_1 + \alpha_2)$

$p_{17}^{(5,6)} = 1$ -$\tilde{H}(\alpha_1)$ - $\frac{\alpha_1}{\alpha_1+\alpha_2}$ + $\frac{\alpha_1}{\alpha_1+\alpha_2}\tilde{H}(\alpha_1 + \alpha_2)$

$p_{20} = \tilde{G}_1(\alpha_2)$

$p_{23}^{(7)} = 1 - \tilde{G}_1(\alpha_2)$

$p_{30} = \tilde{G}_2(\alpha_1)$

$p_{37} = 1 - \tilde{G}_2(\alpha_1)$

$p_{73} = 1$

It can easily verified that

$p_{01} + p_{02} + p_{03} = 1$

$p_{10} + p_{12}^{(4)} + p_{13}^{(5)} + p_{17}^{(4,6)} + p_{17}^{(5,6)} = 1$

$p_{20} + p_{23}^{(7)} = 1$

$p_{30} + p_{37} = 1$

$p_{73} = 1$

(b) The mean sojourn time in various states as follows

$\Psi_0 = \int P(T_0 > t)\, dt = 1/(\alpha_1 + \alpha_2 + \beta)$

$\Psi_1 = [1 - \tilde{H}(\alpha_1 + \alpha_2)]/ (\alpha_1 + \alpha_2)$

$\Psi_2 = [1 - \tilde{G}_1(\alpha_2)] / \alpha_2$

$\Psi_3 = [1 - \tilde{G}_2(\alpha_1)] / \alpha_1$

$\Psi_7 = \int \overline{G}_1(t) dt$

Analysis of Characteristics

(a) Reliability and MTSF

To determine the reliability of the system, when system initially starts from regenerative state S_i, we assume the failed states of the system as absorbing state. Using simple probabilistic arguments in regenerative point technique, the value of $R_0(t)$ in terms of its Laplace transform is

$$R_0^*(s) = \frac{Z_0^* + q_{01}^* Z_1^* + \left(q_{01}^* q_{12}^{(4)*} + q_{02}^*\right) Z_2^* + (q_{01}^* q_{13}^{(5)*} + q_{03}^*) Z_3^*}{1 - q_{01}^* q_{10}^* - q_{01}^* q_{12}^{(4)*} q_{20}^* - q_{01}^* q_{13}^{(5)*} q_{30}^* - q_{02}^* q_{20}^* - q_{03}^* q_{30}^*}$$

where Z_0^*, Z_1^*, Z_2^* and Z_3^* are the L.T. of

$Z_0(t) = e^{-(\alpha_1+\alpha_2+\beta)t}$, $Z_1(t) = e^{-(\alpha_1+\alpha_2)t}$

$Z_2(t) = e^{-\alpha_2 t}\overline{G_1}(t)$, $Z_3(t) = e^{-\alpha_1 t}\overline{G_2}(t)$

The mean time to system failure is given by

$$E(T_0) = \lim_{s \to 0} R_0^*(s) = \frac{N_1(0)}{D_1(0)}$$

$$E(T_0) = \frac{\Psi_0 + p_{01}\Psi_1 + \left(p_{01} p_{12}^{(4)} + p_{02}\right)\Psi_2 + (p_{01} p_{13}^{(5)} + p_{03})\Psi_3}{1 - p_{01}p_{10} - p_{01}p_{12}^{(4)}p_{20} - p_{01}p_{13}^{(5)}p_{30} - p_{02}p_{20} - p_{03}p_{30}}$$

(b) Availability Analysis

Let $A_i(t)$be the probability that the system is up at epoch t, when initially system starts operation from state S_i ƐE. Using the regenerative point technique and the tools of Laplace transforms, one can obtain the values of $A_0(t)$ in terms of their Laplace Transformations i.e. $A_0^*(s)$. Then the steady-state availability of the system is given by

$A_0 = N_1/D_1$

Where,

$$N_1 = p_{30}\Psi_0 + p_{30}p_{01}\Psi_1 + p_{30}(p_{01}p_{12}^{(4)} + p_{02})\Psi_2 + \left(1 - p_{01}p_{10} - p_{02}p_{20} - p_{01}p_{12}^{(4)}p_{20}\right)\Psi_3$$

and

$$D_1 = p_{30}\Psi_0 + p_{30}p_{01}\Psi_1 + p_{30}(p_{01}p_{12}^{(4)} + p_{02})\Psi_2 + \left(1 - p_{01}p_{10} - p_{02}p_{20} - p_{01}p_{12}^{(4)}p_{20}\right)\Psi_3 + (p_{37} - p_{01}p_{10}p_{37} - p_{02}p_{20}p_{37} - p_{01}p_{12}^{(4)}p_{20}p_{37} + p_{01}p_{17}^{(4,6)}p_{30} + p_{01}p_{17}^{(5,6)}p_{30})\Psi_7$$

(c)Busy Period Analysis

(1) Due to First Unit

Let $B_i^I(t)$ be the probability that the repair facility is busy in repair of first unit at epoch t, when initially system starts functioning from state S_iƐE. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $B_i^I(t)$ in terms of their Laplace transform i.e. $B_i^{I*}(s)$. In the long run the steady state probability that the repairman will be busy in repair of unit- I is given by

$$B_0^I = N_2/D_1$$

Where,

$$N_2 = p_{30}(p_{01}p_{12}^{(4)} + p_{02})\Psi_2 + (p_{37} - p_{01}p_{10}p_{37} - p_{02}p_{20}p_{37} - p_{01}p_{12}^{(4)}p_{20}p_{37} + p_{01}p_{17}^{(4,6)}p_{30} + p_{30}p_{01}p_{17}^{(5,6)})\Psi_7$$

The value of D_1 is same as given in (b).

(2)Due to Second Unit

Let $B_i^{II}(t)$ be the probability that the repair facility is busy in repair of second unit at epoch t, when initially system starts functioning from state S_iƐ E. Using the regenerative point technique and the tools of Laplace transform, one can obtain the value of $B_i^{II}(t)$ in terms of their Laplace transform i.e. $B_i^{II*}(s)$. In the long run the steady state probability that the repairman will be busy in repair of unit- II is given by

$$B_0^{II} = N_3/D_1$$

Where,

$$N_3 = \left(1 - p_{01}p_{10} - p_{02}p_{20} - p_{01}\, p_{12}^{(4)}p_{20}\right)\Psi_3$$

The value of D_1 is same as given in (b).

(d Cost Benefit Analysis

Let K_0 is the per unit up time revenue by the system due to operation of any unit. K_1 and K_2 be the repair cost per unit time when unit- I and unit- II will be under repair respectively. Then the net expected profit incurred by the system during time interval (0, t) is given by

$$P(t) = K_0\mu_{up}(t) - K_1\mu_b^I(t) - K_2\mu_b^{II}(t)$$

The expected profit per- unit time in steady-state is

$$P = K_0A_0 - K_1B_0^I - K_2B_0^{II}$$

Conclusion

This paper explain the importance of introducing inspection policy to the system having when the repairman is appear and disappear for obtaining the effectiveness of different reliability measures. Thus the results obtained, provides an effective information and new ideas for new researchers and companies to prefer such conditions for same systems.

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